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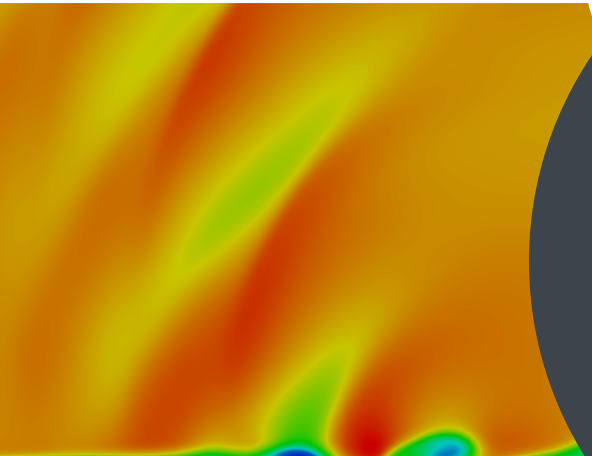


SEAL - Simulation der Entstehung und Ausbreitung von Lärm in Strömungen

7. HPC-Status-Konferenz der Gauß-Allianz

Outline

1. Introduction
2. NISP - Theory
3. Discontinuous Galerkin CFD-Solver
4. Results Cavity Simulation
5. Conclusions and outlook

Introduction

1

SEAL - Project Objective

Motivation

Interaction of flow and noise can cause highly sensitive acoustic feedback loops

Project Goal

"The central goal of SEAL is the numerical simulation of **fluid flows** with time-accurate resolution of **acoustic waves** in **industrially relevant** configurations under **uncertain conditions**."

SEAL - Project Building Blocks

Direct Noise Computation

Multiscale Numerics

- Strong but sensitive coupling of noise and hydrodynamics
- Turbulence: Neutral and stable schemes for Large Eddy Simulation, low interaction with acoustics
- Complex geometries
- Long sampling times

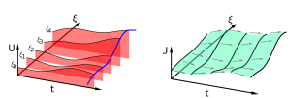
Uncertainty Quantification

Non-Intrusive Methods

- Multilevel Monte Carlo (MLMC)
- Generalized Polynomial Chaos (gPC)

Intrusive Methods

- Stochastic Galerkin (SG)



High Performance Computing

Non-Intrusive Methods

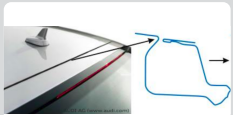
- Optimal scheduling on Hazelhen
- For MLMC: Samples and computing requirements variable during runtime

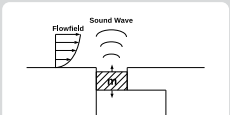
Intrusive Methods

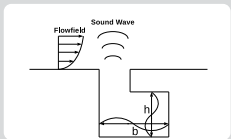
- New parallelization strategy
- Curse of dimensionality: Memory and Computing Time

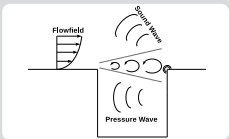
SEAL - Current Status

Aeroacoustics









Non-Intrusive UQ

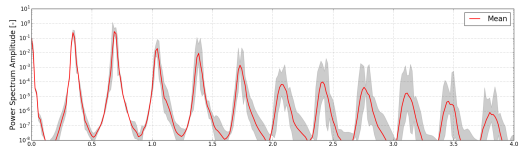
Multi-Level-Monte-Carlo (MLMC):

$$\mathbb{E}(Q(u)) \approx \sum_{l=0}^{L} \frac{1}{N_l} \sum_{k=0}^{N_l} (Q(\hat{u}_l^k) - Q(\hat{u}_{l-1}^k))$$

Polynomial Chaos (gPC):

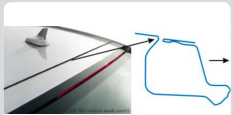
$$u(t, x, \xi(\omega)) = \sum_{p=0}^{\infty} u^p(t, x) \Psi_p(\xi(\omega))$$

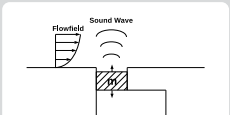
Influence of UQ on Aeroacoustic Solution

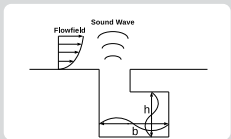


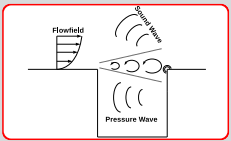
SEAL - Current Status

Aeroacoustics









Non-Intrusive UQ

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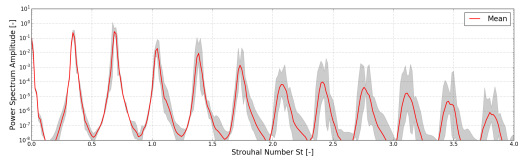
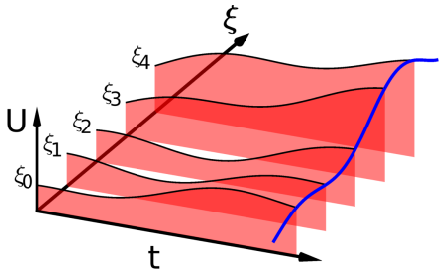


NISP - Theory

Non-Intrusive-Spectral-Projection (NISP)



- Polynomial Chaos Expansion (PCE) of quantity of interest
- $Q(u(x,t,\xi)) = \sum_{i=0}^{\infty} \hat{Q}_i(u(x,t))\Psi_i(\xi)$
- Truncate to N_{Stoch}
- Galerkin projection with associated inner product: $\hat{Q}_i = \frac{\langle Q,\Psi_i\rangle}{\langle \Psi_i,\Psi_i\rangle}$

Advantages

- blackbox solver
- easy to implement
- response surface

Disadvantages

- computationally expensive for multiple stochastic variables
- sparse grid necessary

NISP - Theory

Choose an orthonormal polynomial basis Ψ_i with respect to a specific pdf f:

$$\int_\Omega \Psi_i(\xi)\Psi_j(\xi)f(\xi)d\xi = \delta_{i,j} \tag{1}$$

- for a **uniform distribution** $\Rightarrow$ Legendre-Polynomials
- for a **normal distribution** $\Rightarrow$ Hermite-Polynomials

Employ suitable quadrature rule:

$$\hat{Q}_i = \frac{1}{\gamma_i}\langle Q, \Psi_i\rangle = \frac{1}{\gamma_i}\int_\Omega Q(\xi)\Psi_i(\xi)f(\xi)d\xi \approx \frac{1}{\gamma_i}\sum_{k=1}^{N} Q(\xi^k)\Psi_i(\xi^k)\omega^k \tag{2}$$

Hence the stochastic quantities of interest can be computed by the corresponding quadrature rules, exploiting the orthogonality of the basis:

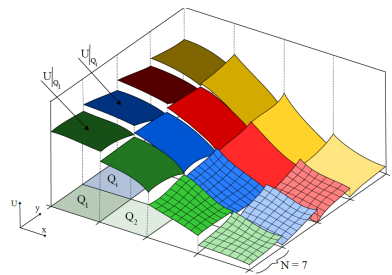
$$\mathbb{E}(Q) \approx \hat{Q}_0 = \sum_{i=1}^{N} Q(\xi^k)\omega_k \tag{3}$$

Discontinuous Galerkin CFD-Solver

3

The discontinuous Galerkin spectral element method code FLEXI

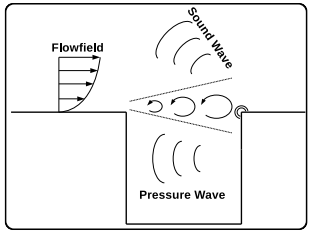
- Hexahedral elements (unstructured)
- Lagrange polynomial basis functions in each element
- Riemann solver for discontinuities at cell interfaces
  $\Rightarrow$ compact stencil, excellent parallelization
- Legendre-Gauss or Legendre-Gauss-Lobatto points for both interpolation and integration (diagonal mass matrix)
- Explicit Runge-Kutta time integration



Results
Cavity Simulation

4

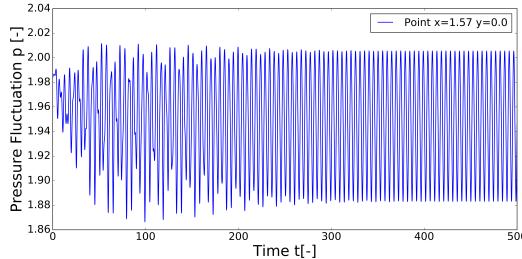
Aeroacoustics of a 2D cavity: Rossiter feedback

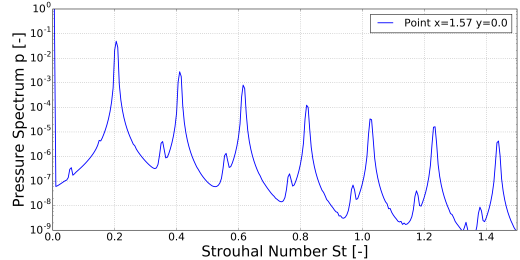


The **numerical setup** from T. Colonius is chosen in order to have a stable and computationally inexpensive 2D simulation, suitable for **UQ simulations**:

Ma	L/D	Boundary Layer	δ_{99}	Re_D
0.6	2	Laminar	$0.28D$	1500

Table: Cavity Setup from T. Colonius (99)





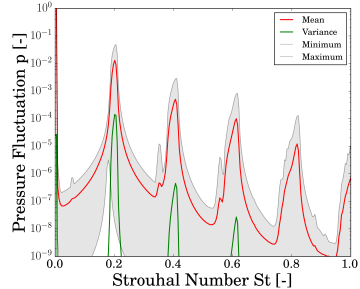
NISP - 2D cavity results

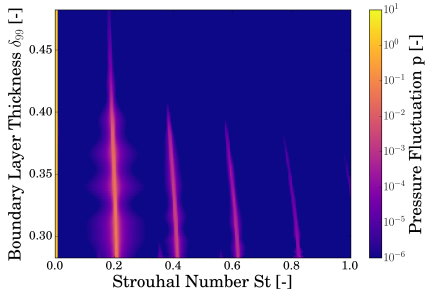
Noise sensitivity due to an uncertain initial laminar boundary layer thickness δ_{99}:

- Frequencies dependent on boundary layer thickness $\Rightarrow$ velocity above the cavity
- Higher amplitudes towards thinner boundary layer $\Rightarrow$ farfield noise increases
- For this configuration the critical boundary layer thickness for Rossiter feedback is $\delta_{99} \approx 0.5$

NISP simulation setup:

N_ξ	Distribution	UQ Parameter	Range
25	Uniform	BL Thickness	$0.28 - 0.48$





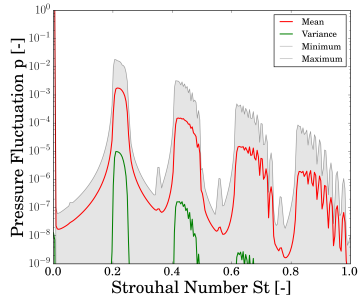
NISP - 2D cavity results

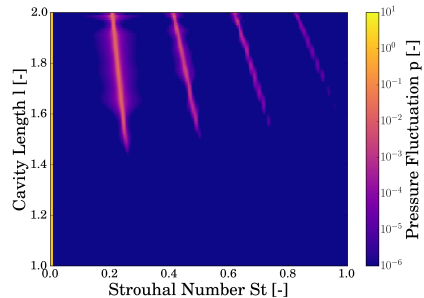
Noise sensitivity due to an uncertain cavity length l:

- Frequencies dependent on cavity length $\Rightarrow$ Rossiter feedback formula
- For this configuration the critical cavity length l for Rossiter feedback is $l \approx 1.5$
- Stochastic resolution N_ξ is to low in order to resolve mean and variance

NISP simulation setup:

N_ξ	Distribution	UQ Parameter	Range l
25	Uniform	Cavity Length l	$1.0 - 2.0$





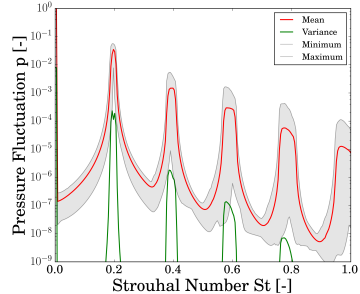
NISP - 2D cavity results

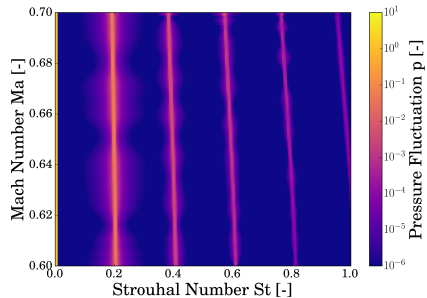
Noise sensitivity due to an uncertain inflow Mach number Ma: ($\delta_{99} = 0.31$)

- Frequencies dependent on inflow Mach Number or flow speed
- Towards higher harmonics, higher dependency on inflow Mach number $\Rightarrow$ both expected from Rossiter's formula
- Higher amplitudes towards higher inflow Mach numbers $\Rightarrow$ farfield noise increases

NISP simulation setup:

N_ξ	Distribution	UQ Parameter	Range Ma
25	Uniform	Mach number	$0.6 - 0.7$

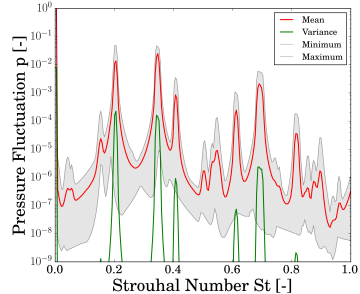




NISP - 2D cavity results

Noise sensitivity due to an uncertain inflow Mach number Ma: ($\delta_{99} = 0.28$)

- Frequencies dependent on inflow Mach number
- For this configuration the dominant Rossiter mode depends on the inflow Mach number
- This behaviour is **not predicted** by Rossiter estimates!
- Further investigations with $N_{dim(stoch)} = 2$ (BL thickness and Mach number)



NISP simulation setup:

N_ξ	Distribution	UQ Parameter	Range Ma
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Aeroacoustics of a 3D cavity

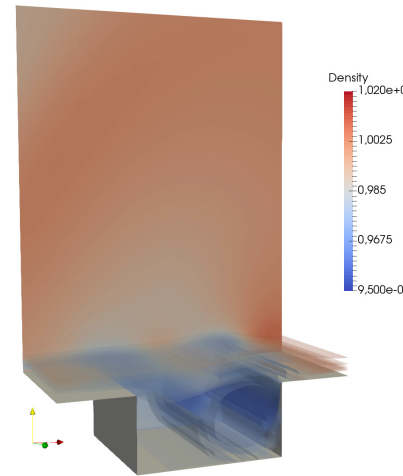
Simulation setup:

Ma	L/D	Boundary Layer	δ_{99}	Re_D
0.6	2	Laminar	$0.28D$	1500

Table: Cavity setup from T. Colonius extend to 3D (99)

Results:

- Up to $t \approx 330$ only 2D Rossiter feedback, then a 3D instability develops
- A standing wave in z-direction and a Rossiter feedback exist simultaneously
- Energy transfer from feedback into crosswave observable



CPU-Time for Cavity NISP simulations

	Cells	N	DoFs	DoFs/Core	Cores	CPUh
2D	1500	5	54000	3000	24	40
3D	7500	5	1,6 Mio.	3000	528	880

Table: Computational cost for 2D/3D deterministic simulation

2D		NStoch		
		5	10	20
nDim Stoch.	1	200	400	800
	2	1000	4000	16000
	3	5000	40000	320000

Table: Computational cost in CPUh for 2D setup with up to nDim=3 stochastic dim.

3D		NStoch		
		5	10	20
nDim Stoch.	1	4400	8800	17600
	2	22000	88000	352000
	3	110000	880000	7,0 Mio.

Table: Computational cost in CPUh for 3D setup with up to nDim=3 stochastic dim.

Conclusions and outlook

5

Conclusions and outlook

Conclusion:

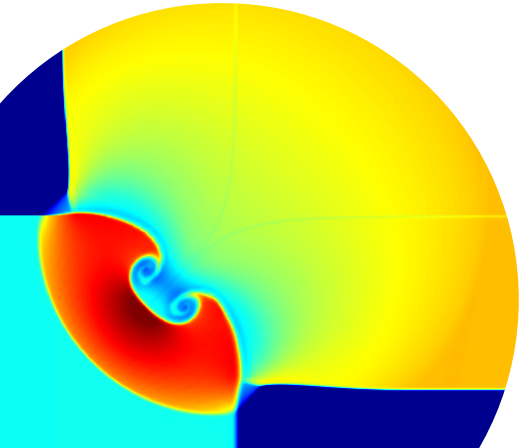
- a NISP and an MLMC framework for aeroacoustic applications has been established (MLMC not shown here)
- both frameworks are validated against a Monte Carlo simulation
- NISP has been applied to a 2D cavity problem with Rossiter feedback and has revealed some interesting features!

Outlook:

- a comparison of MLMC and NISP for multiple stochastic input parameters
- simulation of a 3D turbulent cavity problem and validation against experimental results
- apply NISP to 3D turbulent cavity problems
- development of a 3D intrusive Stochastic-Galerkin solver

flexi-project.org

Thank you for your attention!



Baden-Württemberg Stiftung
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