Improvements to ELPA Eigensolvers

ELPA-AEO

http://elpa-aeo.mpcdf.mpg.de

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Project Partners

Lehrstuhl für Angewandte Informatik
Prof. B. Lang (BUW)

Prof. M. Scheffler, Dr. Ch. Carbogno (FHI)

Dr. H. Lederer, Dr. A. Marek (MPCDF)

Lehrstuhl für Informatik mit Schwerpunkt Wissenschaftliches Rechnen (TUM-SCCS)
Prof. H.-J. Bungartz, Prof. Th. Huckle

Lehrstuhl für Theoretische Chemie
Prof. K. Reuter, Dr. Ch. Scheurer (TUM-CH)
Stepwise Approach for the Eigenproblem

Solution of the generalized matrix eigenproblem generally proceeds in five steps:

(I) Transformation to a dense standard eigenproblem (e.g., by Cholesky decomposition of S),
\[ H_{KS}c_i = \varepsilon_i Sc_i \rightarrow Aq_A = \lambda q_A, \quad \lambda = \varepsilon_i \]

(II) Reduction to tridiagonal form, \( A \rightarrow T \);

(III) Solution of the tridiagonal problem for \( k \) eigenvalues and -vectors, \( Tq_T = \lambda q_T \);

(IV) Back transformation of \( k \) eigenvectors to dense orthonormal form, \( q_T \rightarrow q_A \);

(V) Back transformation to the original, non-orthonormal basis, \( q_A \rightarrow c_i \).
ELPA1: one-step direct solver

Pros: allows full use of matrix–matrix products and sparse matrix-vector products
Cons: gives rise to one extra back transformation step of the eigenvectors

ELPA2: two-step direct solver

Pros: allows full use of matrix–matrix products and sparse matrix-vector products
Cons: gives rise to one extra back transformation step of the eigenvectors
ELPA Performance Gain in 2017
Algorithmic improvements / code optimizations

System: Hydra (node: 2 x Intel Xeon E5-2680v2, 20 cores)
No of cores used: 80
Matrix size: 20,000, real
Solver: ELPA2, 100% Eigenvectors

ELPA 2013 -> ELPA 2017:
Runtime (double precision) on 80 cores: ~75 s -> 50 s
Improvement factor ~ 1.5

Option for mixing precision (both single and double precision supported):
ELPA 2017
Run time (double precision) 50 s -> 30 s (single precision)
Improvement factor ~ 1.7
Monitoring

Monitoring tool for autotuning fully implemented
Detailed timing measurements (both exclusive and inclusive) realized for all major routines (elpa1 solver, elpa2 solver, elpa2 optimized kernels, transformation of generalized to standard problem incl. its back transformation

Monitoring for reporting to calling program
Calling program can
- enable/disable timing measurements
- query timing results, number of calls of a subroutine and performance in GigaFlop/s
- print timing results to stdout
- for autotuning
For a given problem hardware configuration:

Automatic selection of the most efficient procedure for the solution of the eigenproblem

For a given problem (matrix size and number of eigenvalues needed):

Comparing the different available procedures (elpa1 or elpa2) in consecutive SCF cycles, identification and automatic selection of the most efficient one for all further steps.

For elpa2 (more complex than elpa1):

Optimization of the selection of the block size and the different compute-intensive kernels. In case of availability of GPUs, inclusion into selection criteria.
Autotuning

Based on monitoring infrastructure

Main program options: enable/disable (default: disable)

In case of enable:

When elpa is called multiple times within an SCF iteration, different elpa solvers (elpa 1 and elpa 2), different tuning options for elpa 2 (kernel) and further run time parameters are tried consecutively and the timings recorded. After measuring all available options, the best performing one is used for all subsequent calls.

User guided autotuning:

User can preselect a subset of settings (e.g. elpa2), then the other options are not considered.
User results with autotuning

Accumulated Runtimes:

ZrO$_2$ – 96 atoms
8 Intel Xeon E5-2698v3 CPUs
(4 nodes, 32 cores/node)

accumulated run-time vs. SCF cycles

No Autotuning vs. Autotuning
Improvements to the generalized eigenvalue problem $AX = BX\mu$

<table>
<thead>
<tr>
<th>Mathematics</th>
<th>What ELPA provides</th>
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<tbody>
<tr>
<td>1) $B = U^H U$ (Cholesky decomposition)</td>
<td>1a) $B = U^H U$ (Cholesky decomposition)</td>
</tr>
<tr>
<td></td>
<td>1b) Explizit construction of inverse $U^{-1}$</td>
</tr>
<tr>
<td>2) $A \rightarrow A^* = U^{-H} A U^{-1}$ (transformation to standard eigenproblem)</td>
<td>2) identical</td>
</tr>
<tr>
<td>3) solve standard eigenvalue problem</td>
<td>3) identical</td>
</tr>
<tr>
<td>4) Backtransformation of eigenvectors according to 2</td>
<td>4) identical</td>
</tr>
</tbody>
</table>

New: 

```
elpa_generalized
combining steps 1a – 4 in single routine
```
Results

Improvements to the generalized eigenvalue problem

\[ AX = BX\mu \]
Results

Improvements to the generalized eigenvalue problem

\[ AX = BX\mu \]

Latest ELPA release:

Under certain conditions, an alternative, faster implementation is used
Supported Architectures

- **X86 up to Intel SkyLake / AVX512**
  measured up to matrix size 0.5 M up to 40k cores

- **X86 / Intel Xeon KNL**
  measured up to matrix size 1 M on 200k cores

- **IBM Power 8+9**

- **GPU systems**
  a) nVIDIA K20X/K40 in X86 host with via PCIe
  b) nVIDIA P100 with Power8 hosts via NVLINK1
  c) GTX1080 with X86 host via PCIe

- **K Computer** (RIKEN)

- Porting started:
  **NEC SX-Aurora** vector processors
  **OpenPower:** NVIDIA V100 GPUs in Power 9 hosts
  NVIDIA V100 GPUs in Intel Xeon CPU hosts

(also running on ARM and on MAC OS systems)
Presentations

P. Kus:
GPU Optimization of Large-Scale Eigenvalue Solver, ENUMATH 2017

B. Lang:
The ESSEX-II and ELPA-AEO Projects. EPASA2018
Int'l Workshop on Eigenvalue Problems: Algorithms; Software and Applications, in Petascale Computing, March 5-6, 2018, Tsukuba

B. Lang, V. Manin:
Reduction of generalized HPD eigenproblems using Cannon's algorithm.
SIAM Conference on Parallel Processing for Scientific Computing, March 7-10, 2018, Tokyo


PMAA18: Minisymposium on eigenvalue problems and applications, organised by Th. Huckle (TUM), B. Lang (BUW), and T. Imamura (RIKEN)
Presentations from: BUW, TUM-SCSC, FHI, MPCDF

Conference: Solving or Circumventing Eigenvalue Problems in Electronic Structure Theory, August 15-17, 2018, Richmond, VA
B. Lang (BUW): New algorithmic developments in ELPA-AEO
C. Carbogno (FHI): Recent Advancements in ELPA: Best Practices in Real Applications

H. Lederer, MPCDF
B. Lang and V. Manin:
Cannon-type triangular matrix multiplication for the reduction of generalized HPD eigenproblems to standard form, Parallel Computing 2018

B. Lang:
Efficient reduction of banded hpd generalized eigenvalue problems to standard form

Benefits from using mixed precision computations in the ELPA-AEO and ESSEX-II eigensolver projects

B. Lang
New algorithmic developments in ELPA-AEO
Solving or Circumventing Eigenvalue Problems in Electronic Structure Theory, August 15-17, 2018, Richmond, VA
New ELPA Releases

ELPA-Release from May 2018

- Extended autotuning of performance critical parameters (as cache blocking, workload distribution between CPU and GPU)
- Driver routines for generalized problems
- Option to skip decomposition of B matrix of generalized eigenproblem

ELPA-Release planned for Nov 2018

- Automatic checkpoint/restart for autotuning
  -> Option to start new simulation with restart file from previous autotuning run
- Improved transformation from generalized to standard problem