

# Improvements to ELPA Eigensolvers

ELPA-AEO

<http://elpa-aeo.mpcdf.mpg.de>

GEFÖRDERT VOM



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und Forschung

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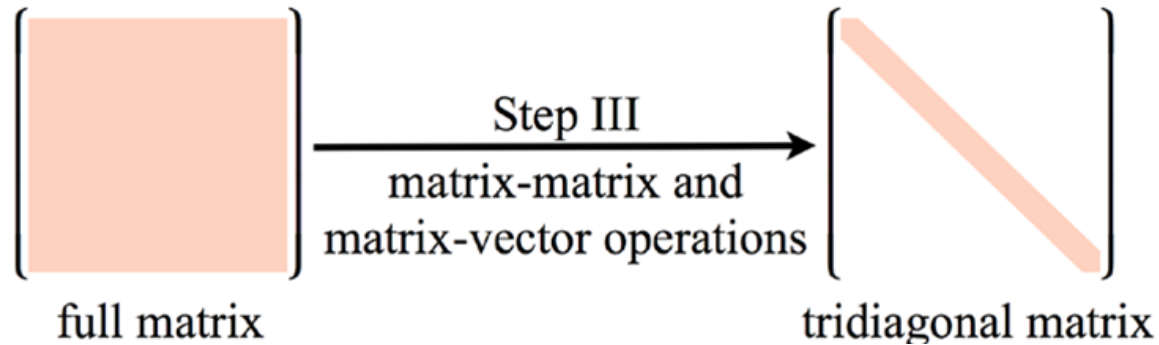
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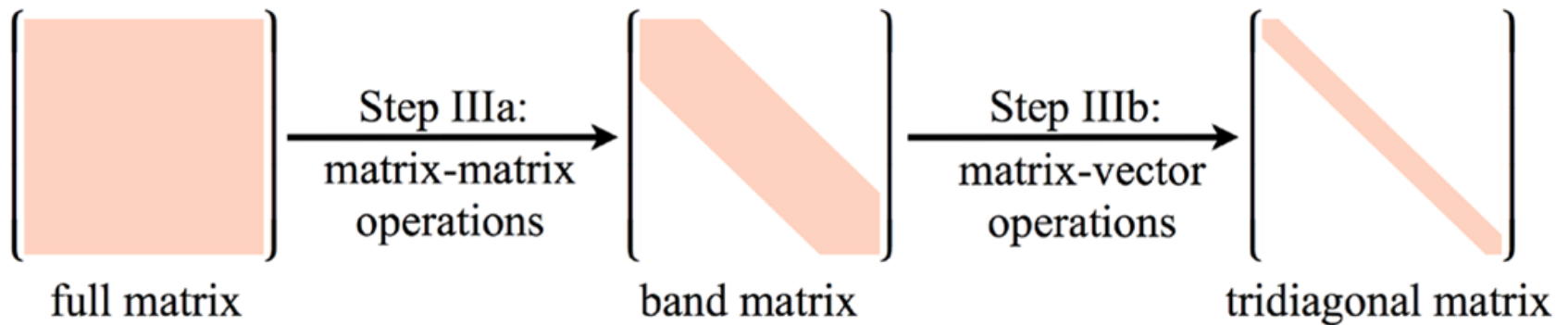
**Solution of the generalized matrix eigenproblem generally proceeds in five steps:**

- (I) Transformation to a dense standard eigenproblem (e.g., by Cholesky decomposition of  $S$ ),  
 $H_{KS}c_I = \varepsilon_I S c_I \rightarrow Aq_A = \lambda q_A, \quad \lambda = \varepsilon_I$
- (II) Reduction to tridiagonal form,  $A \rightarrow T$ ;
- (III) Solution of the tridiagonal problem for  $k$  eigenvalues and -vectors,  $Tq_T = \lambda q_T$ ;
- (IV) Back transformation of  $k$  eigenvectors to dense orthonormal form,  $q_T \rightarrow q_A$ ;
- (V) Back transformation to the original, non-orthonormal basis,  $q_A \rightarrow c_I$ .

## ELPA1: one-step direct solver



## ELPA2: two-step direct solver



**Pros:** allows full use of matrix–matrix products and sparse matrix-vector products

**Cons:** gives rise to one extra back transformation step of the eigenvectors

Algorithmic improvements / code optimizations

System:	Hydra (node: 2 x Intel Xeon E5-2680v2, 20 cores)
No of cores used:	80
Matrix size:	20 000, real
Solver:	ELPA2, 100% Eigenvectors

## **ELPA 2013 -> ELPA 2017:**

Runtime (double precision) on 80 cores: ~ **75 s -> 50 s**

**Improvement factor ~ 1.5**

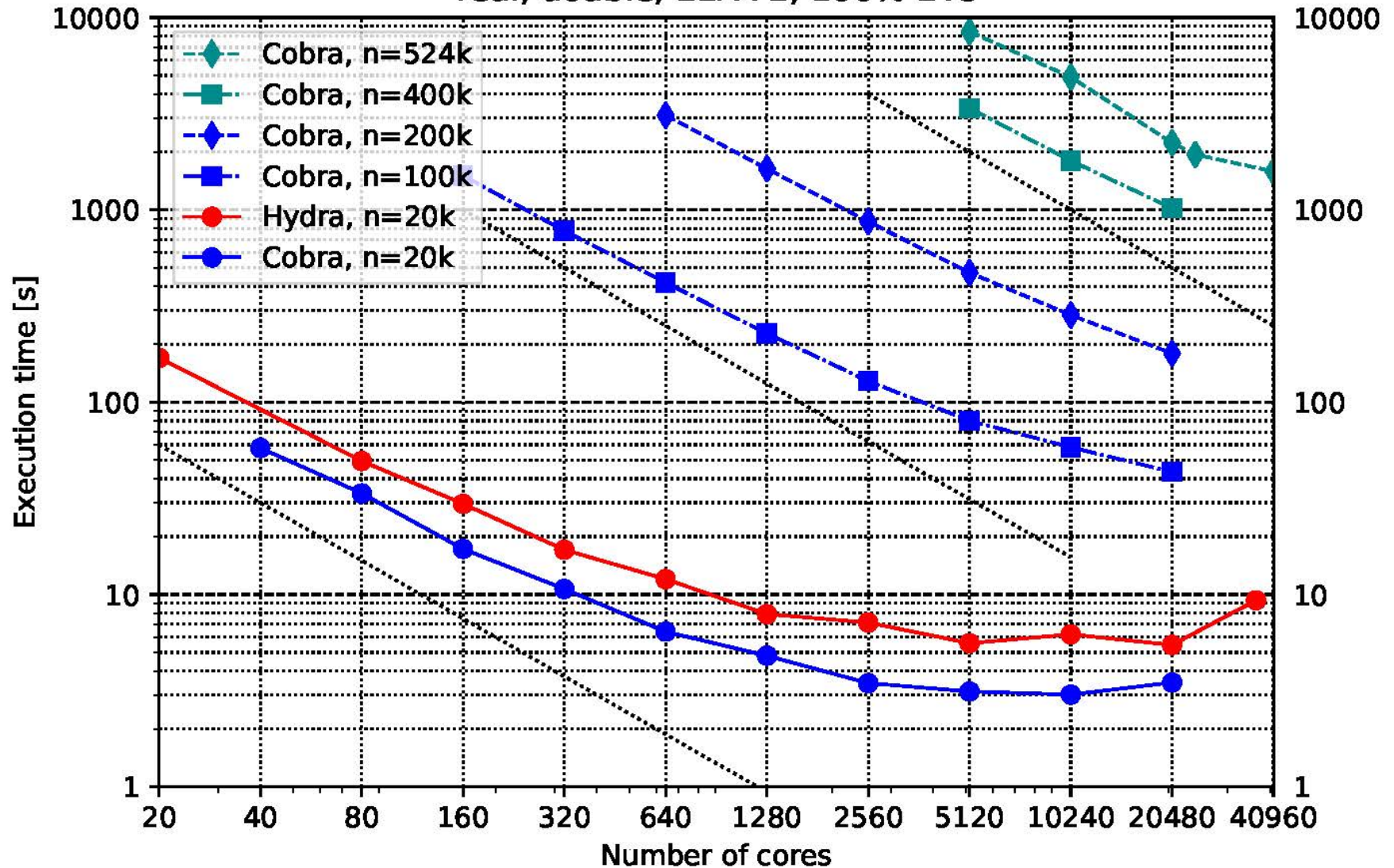
Option for mixing precision (both single and double precision supported):

## **ELPA 2017**

Run time (double precision ) **50 s -> 30 s** ( single precision)

**Improvement factor ~ 1.7**

real, double, ELPA 2, 100% EVs



## Monitoring tool for autotuning fully implemented

Detailed timing measurements (both exclusive and inclusive) realized for all major routines (elpa1 solver, elpa2 solver, elpa2 optimized kernels, transformation of generalized to standard problem incl. its back transformation)

## Monitoring for reporting to calling program

Calling program can

- enable/disable timing measurements
- query timing results, number of calls of a subroutine and performance in GigaFlop/s
- print timing results to stdout
- for autotuning

# Improvements by Automatic ELPA Autotuning

## For a given problem hardware configuration:

Automatic selection of the most efficient procedure for the solution of the eigenproblem

## For a given problem (matrix size and number of eigenvalues needed):

Comparing the different available procedures (elpa1 or elpa2) in consecutive SCF cycles, identification and automatic selection of the most efficient one for all further steps.

## For elpa2 (more complex than elpa1) :

Optimization of the selection of the block size and the different compute-intensive kernels.

In case of availability of GPUs, inclusion into selection criteria.



## Based on monitoring infrastructure

Main program options: enable/disable (default: disable)

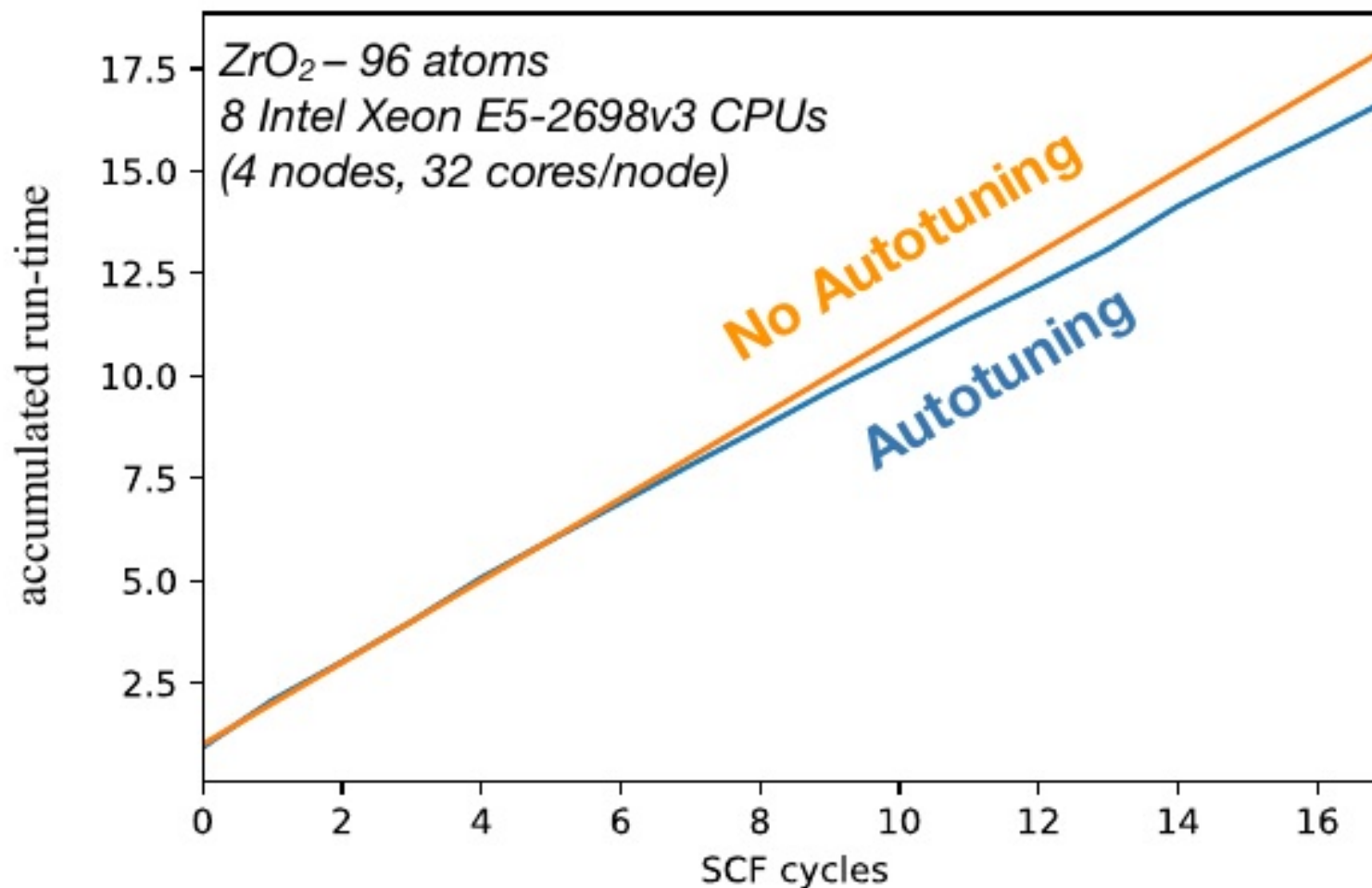
### In case of enable:

When elpa is called multiple times within an SCF iteration, different elpa solvers (elpa 1 and elpa 2), different tuning options for elpa 2 (kernel) and further run time parameters are tried consecutively and the timings recorded. After measuring all available options, the best performing one is used for all subsequent calls .

### User guided autotuning:

User can preselect a subset of settings (e.g. elpa2), then the other options are not considered.

## Accumulated Runtimes:



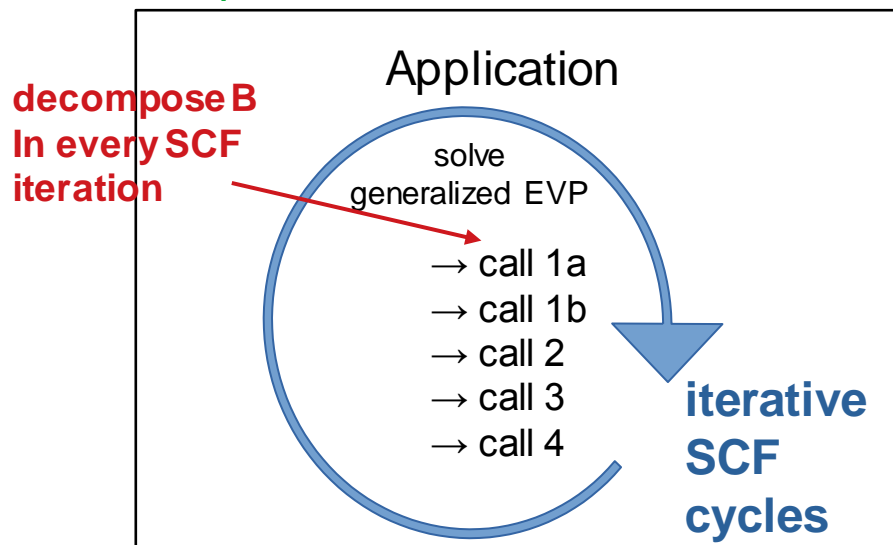
# Improvements to the generalized eigenvalue problem $AX = BX\mu$

Mathematics	What ELPA provides
1) $B = U^H U$ (Cholesky decomposition)  2) $A \rightarrow A^* = U^{-H} A U^{-1}$ (transformation to standard eigenproblem)  3) solve standard eigenvalue problem  4) Backtransformation of eigenvectors according to 2	1a) $B = U^H U$ (Cholesky decomposition) 1b) Explizit construction of inverse $U^{-1}$  2) identical  3) identical  4) identical  <p style="text-align: right;">New:  <b>elpa_generalized</b>            combining steps 1a – 4            in single routine</p>

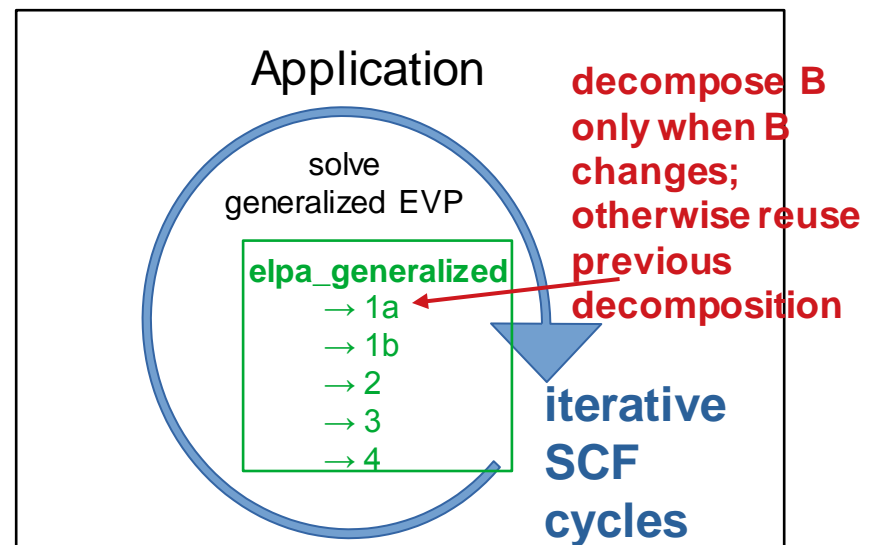
# Results

Improvements to the generalized eigenvalue problem  
 $AX = BX\mu$

Up to last ELPA release in 2017:

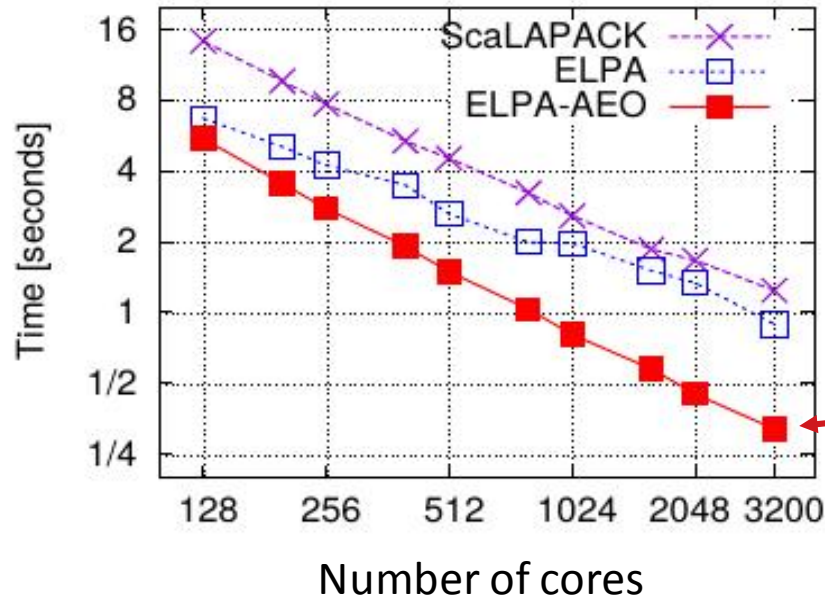


Latest ELPA release:

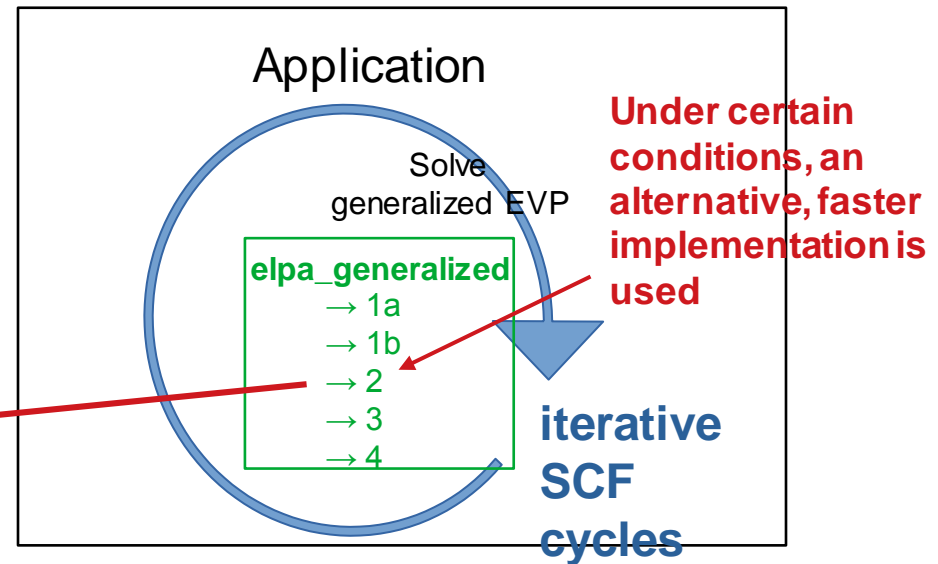


# Results

Improvements to the generalized eigenvalue problem  
 $AX = BX\mu$



Latest ELPA release:



- **X86 up to Intel SkyLake / AVX512**  
measured up to matrix size 0.5 M up to 40k cores
- **X86 / Intel Xeon KNL**  
measured up to matrix size 1 M on 200k cores
- **IBM Power 8+9**
- **GPU systems**
  - a) nVIDIA K20X/K40 in X86 host with via PCIe
  - b) nVIDIA P100 with Power8 hosts via NVLINK1
  - c) GTX1080 with X86 host via PCIe
- **K Computer (RIKEN)**
- Porting started:
  - NEC SX-Aurora** vector processors
  - OpenPower:** NVIDIA V100 GPUs in Power 9 hosts
  - NVIDIA V100 GPUs** in Intel Xeon CPU hosts

*(also running on ARM and on MAC OS systems)*

P. Kus:

GPU Optimization of Large-Scale Eigenvalue Solver, ENUMATH 2017

B. Lang:

The ESSEX-II and ELPA-AEO Projects. EPASA2018

Int'l Workshop on Eigenvalue Problems: Algorithms; Software and Applications, in Petascale Computing, March 5-6, 2018, Tsukuba

B. Lang, V. Manin:

Reduction of generalized HPD eigenproblems using Cannon's algorithm.

SIAM Conference on Parallel Processing for Scientific Computing, March 7-10, 2018, Tokyo

B. Lang, V. Manin: Efficient reduction of generalized HPD eigenproblems. PMAA18 { 10th Int'l Workshop on Parallel Matrix Algorithms and Applications, 27.-29. Juni 2018, Zürich

PMAA18: Minisymposium on eigenvalue problems and applications, organised by Th. Huckle (TUM), B. Lang (BUW), and T. Imamura (RIKEN)

Presentations from: BUW, TUM-SCSC, FHI, MPCDF

Conference: Solving or Circumventing Eigenvalue Problems in Electronic Structure Theory, August 15-17, 2018, Richmond, VA

B. Lang (BUW): New algorithmic developments in ELPA-AEO

C. Carbogno (FHI): Recent Advancements in ELPA: Best Practices in Real Applications

B. Lang and V. Manin:

Cannon-type triangular matrix multiplication for the reduction of generalized HPD eigenproblems to standard form, *Parallel Computing* 2018

B. Lang:

**Efficient reduction of banded hpd generalized eigenvalue problems to standard form**

A. Alvermann, A. Basermann, H.-J. Bungartz, Ch. Carbogno, D. Ernst, H. Fehske, Y. Futamura, M. Galgon, G. Hager, S. Huber, Th. Huckle, A. Ida, A. Imakura, S. Köcher, M. Kreutzer, P. Kus, B. Lang, H. Lederer, V. Manin, A. Marek, K. Nakajima, L. Nemec, K. Reuter, M. Rippl, M. Röhrig-Zöllner, T. Sakurai, M. Scheer, Ch. Scheurer, F. Shahzad, D. Simoes Brambila, J. Thies, G. Wellein:

**Benefits from using mixed precision computations in the ELPA-AEO and ESSEX-II eigensolver projects**

B. Lang

New algorithmic developments in ELPA-AEO

**Solving or Circumventing Eigenvalue Problems in Electronic Structure Theory, August 15-17, 2018, Richmond, VA**



## ELPA-Release from May 2018

- Extended autotuning of performance critical parameters (as cache blocking, workload distribution between CPU and GPU)
- Driver routines for generalized problems
- Option to skip decomposition of B matrix of generalized eigenproblem

## ELPA-Release planned for Nov 2018

- Automatic checkpoint/restart for autotuning  
-> Option to start new simulation with restart file from previous autotuning run
- Improved transformation from generalized to standard problem